Assess

Exercises 1–7 Be sure students understand that while a term such as *segment* has different meanings in the two geometries, other terms such as *intersect*, *parallel*, and *perpendicular* will have the same meaning in each geometry.

Exercise 2 Students should understand that a line segment in spherical geometry must be a part of a line, so it would be a part of a great circle.

Study Notebook

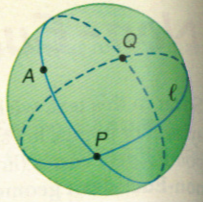
Ask students to summarize what they have learned about how points, lines, and planes in spherical geometry are similar to and different from those terms in Euclidean geometry.

Answers

1. The great circle is finite.
2. A curved path on the great circle passing through two points is the shortest distance between the two points.
3. There exist no parallel lines.
4. Two distinct great circles intersect in exactly two points.
5. A pair of perpendicular great circles divides the sphere into four finite congruent regions.
6. There exist no parallel lines.
7. There are two distances between two points.
8. true
9. False; in spherical geometry, if three points are collinear, any point can be between the other two.
10. False; in spherical geometry, there are no parallel lines.

Geometry Activity

Every great circle of a sphere intersects all other great circles on that sphere in exactly two points. In the figure at the right, one possible line through point A intersects line ℓ at P and Q.

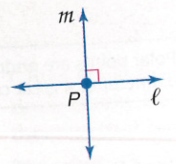


If two great circles divide a sphere into four congruent regions, the lines are perpendicular to each other at their intersection points. Each longitude circle on Earth intersects the equator at right angles.

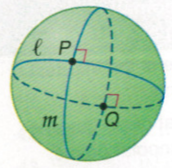
Example *Compare Plane and Spherical Geometries*

For each property listed from plane Euclidean geometry, write a corresponding statement for spherical geometry.

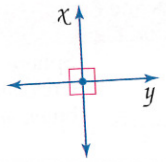
a. Perpendicular lines intersect at one point.



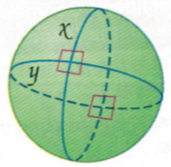
Perpendicular great circles intersect at two points.



b. Perpendicular lines form four right angles.



Perpendicular great circles form eight right angles.



Exercises

For each property from plane Euclidean geometry, write a corresponding statement for spherical geometry. **1–7. See margin.**

1. A line goes on infinitely in two directions.
2. A line segment is the shortest path between two points.
3. Two distinct lines with no point of intersection are parallel.
4. Two distinct intersecting lines intersect in exactly one point.
5. A pair of perpendicular straight lines divides the plane into four infinite regions.
6. Parallel lines have infinitely many common perpendicular lines.
7. There is only one distance that can be measured between two points.

If spherical points are restricted to be nonpolar points, determine if each statement from plane Euclidean geometry is also *true* in spherical geometry. If *false*, explain your reasoning. **8–10. See margin.**

8. Any two distinct points determine exactly one line.
9. If three points are collinear, exactly one point is between the other two.
10. Given a line ℓ and point P not on ℓ, there exists exactly one line parallel to ℓ passing through P.

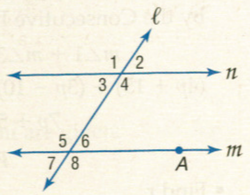
Study Guide and Review

Vocabulary and Concept Check

alternate exterior angles (p. 128)
alternate interior angles (p. 128)
consecutive interior angles (p. 128)
corresponding angles (p. 128)
equidistant (p. 160)
non-Euclidean geometry (p. 165)
parallel lines (p. 126)
parallel planes (p. 126)
plane Euclidean geometry (p. 165)
point-slope form (p. 145)
rate of change (p. 140)
skew lines (p. 127)
slope (p. 139)
slope-intercept form (p. 145)
spherical geometry (p. 165)
transversal (p. 127)

A complete list of postulates and theorems can be found on pages R1–R8.

Exercises **Refer to the figure and choose the term that best completes each sentence.**



1. Angles 4 and 5 are (*consecutive*, *alternate*) interior angles.
2. The distance from point A to line n is the length of the segment (*perpendicular*, *parallel*) to line n through A.
3. If $\angle 4$ and $\angle 6$ are supplementary, lines m and n are said to be (*parallel*, *intersecting*) lines.
4. Line ℓ is a (*slope-intercept*, *transversal*) for lines n and m.
5. $\angle 1$ and $\angle 8$ are (*alternate interior*, *alternate exterior*) angles.
6. If $n \parallel m$, $\angle 6$ and $\angle 3$ are (*supplementary*, *congruent*).
7. Angles 5 and 3 are (*consecutive*, *alternate*) interior angles.

Lesson-by-Lesson Review

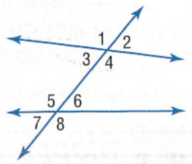
3-1 Parallel Lines and Transversals

See pages 126–131.

Concept Summary

- Coplanar lines that do not intersect are called *parallel*.
- When two lines are cut by a transversal, there are many angle relationships.

Example Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.



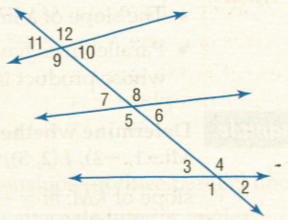
a. $\angle 7$ and $\angle 3$
corresponding

b. $\angle 4$ and $\angle 6$
consecutive interior

c. $\angle 7$ and $\angle 2$
alternate exterior

d. $\angle 3$ and $\angle 6$
alternate interior

Exercises Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles. *See Example 3 on page 128.*



8. $\angle 10$ and $\angle 6$ **corr.**
9. $\angle 5$ and $\angle 12$ **alt. ext.**
10. $\angle 8$ and $\angle 10$ **cons. int.**
11. $\angle 1$ and $\angle 9$ **corr.**
12. $\angle 3$ and $\angle 6$ **alt. int.**
13. $\angle 5$ and $\angle 3$ **cons. int.**
14. $\angle 2$ and $\angle 7$ **alt. ext.**
15. $\angle 8$ and $\angle 9$ **alt. int.**

www.geometryonline.com/vocabulary_review

FOLDABLES Study Organizer

For more information about Foldables, see *Teaching Mathematics with Foldables.*

Have students look through the index cards they added to their Foldables while studying Chapter 3.

Have them edit and/or combine information on the cards as necessary. Remind students to include algebraic examples as well as geometry examples in their notes.

Encourage students to refer to their Foldables while completing the Study Guide and Review and to use them in preparing for the Chapter Test.

Vocabulary and Concept Check

- This alphabetical list of vocabulary terms in Chapter 3 includes a page reference where each term was introduced.
- **Assessment** A vocabulary test/review for Chapter 3 is available on p. 174 of the *Chapter 3 Resource Masters.*

Lesson-by-Lesson Review

For each lesson,

- the main ideas are summarized,
- additional examples review concepts, and
- practice exercises are provided.

Vocabulary PuzzleMaker

ELL The Vocabulary PuzzleMaker software improves students' mathematics vocabulary using four puzzle formats—crossword, scramble, word search using a word list, and word search using clues. Students can work on a computer screen or from a printed handout.

MindJogger Videoquizzes

ELL MindJogger Videoquizzes provide an alternative review of concepts presented in this chapter. Students work in teams in a game show format to gain points for correct answers. The questions are presented in three rounds.

Round 1 Concepts (5 questions)
Round 2 Skills (4 questions)
Round 3 Problem Solving (4 questions)

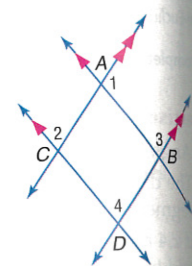
3-2 Angles and Parallel Lines

See pages 133–138.

Concept Summary

- Pairs of congruent angles formed by parallel lines and a transversal are corresponding angles, alternate interior angles, and alternate exterior angles.
- Pairs of consecutive interior angles are supplementary.

Example In the figure, $m\angle 1 = 4p + 15$, $m\angle 3 = 3p - 10$, and $m\angle 4 = 6r + 5$. Find the values of p and r.



- Find p.

Since $\overline{AC} \parallel \overline{BD}$, $\angle 1$ and $\angle 3$ are supplementary by the Consecutive Interior Angles Theorem.

$m\angle 1 + m\angle 3 = 180$ Definition of supplementary angles

$(4p + 15) + (3p - 10) = 180$ Substitution

$7p + 5 = 180$ Simplify.

$p = 25$ Solve for p.

- Find r.

Since $\overline{AB} \parallel \overline{CD}$, $\angle 4 \cong \angle 3$ by the Corresponding Angles Postulate.

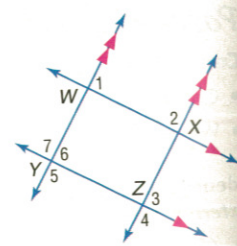
$m\angle 4 = m\angle 3$ Definition of congruent angles

$6r + 5 = 3(25) - 10$ Substitution, $p = 25$

$6r + 5 = 65$ Simplify.

$r = 10$ Solve for x.

Exercises In the figure, $m\angle 1 = 53$. Find the measure of each angle. *See Example 1 on page 133.*



16. $\angle 2$ 127
17. $\angle 3$ 53
18. $\angle 4$ 127
19. $\angle 5$ 127
20. $\angle 6$ 53
21. $\angle 7$ 127
22. In the figure, $m\angle 1 = 3a + 40$, $m\angle 2 = 2a + 25$, and $m\angle 3 = 5b - 26$. Find a and b. *See Example 3 on page 135.* $a = 23$, $b = 27$

3-3 Slopes of Lines

See pages 139–144.

Concept Summary

- The slope of a line is the ratio of its vertical rise to its horizontal run.
- Parallel lines have the same slope, while perpendicular lines have slopes whose product is −1.

Example Determine whether $\overleftrightarrow{KM}$ and $\overleftrightarrow{LN}$ are *parallel*, *perpendicular*, or *neither* for $K(-3, 3)$, $M(-1, -3)$, $L(2, 5)$, and $N(5, -4)$.

slope of $\overleftrightarrow{KM}$: $m = \frac{-3 - 3}{-1 - (-3)}$ or -3 slope of $\overleftrightarrow{LN}$: $m = \frac{-4 - 5}{5 - 2}$ or -3

The slopes are the same. So $\overleftrightarrow{KM}$ and $\overleftrightarrow{LN}$ are parallel.

Exercises Determine whether $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are *parallel*, *perpendicular*, or *neither*. *See Example 3 on page 141.* **23–26. See margin.**

23. $A(-4, 1)$, $B(3, -1)$, $C(2, 2)$, $D(0, 9)$
24. $A(6, 2)$, $B(2, -2)$, $C(-1, -4)$, $D(5, 2)$
25. $A(1, -3)$, $B(4, 5)$, $C(1, -1)$, $D(-7, 2)$
26. $A(2, 0)$, $B(6, 3)$, $C(-1, -4)$, $D(3, -1)$

Graph the line that satisfies each condition. *See Example 4 on page 141.*

27. contains (2, 3) and is parallel to $\overleftrightarrow{AB}$ with $A(-1, 2)$ and $B(1, 6)$
28. contains (−2, −2) and is perpendicular to $\overleftrightarrow{PQ}$ with $P(5, 2)$ and $Q(3, -4)$

27–28. See margin.

3-4 Equations of Lines

See pages 145–150.

Concept Summary

In general, an equation of a line can be written if you are given:

- slope and the y-intercept
- the slope and the coordinates of a point on the line, or
- the coordinates of two points on the line.

Example Write an equation in slope-intercept form of the line that passes through (2, −4) and (−3, 1).

Find the slope of the line.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope Formula

$= \frac{1 - (-4)}{-3 - 2}$ $(x_1, y_1) = (2, -4)$, $(x_2, y_2) = (-3, 1)$

$= \frac{5}{-5}$ or -1 Simplify.

Now use the point-slope form and either point to write an equation.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - (-4) = -1(x - 2)$ $m = -1$, $(x_1, y_1) = (2, -4)$

$y + 4 = -x + 2$ Simplify.

$y = -x - 2$ Subtract 4 from each side.

Exercises Write an equation in slope-intercept form of the line that satisfies the given conditions. *See Examples 1–3 on pages 145 and 146.* **29–34. See margin.**

29. $m = 2$, contains (1, −5)
30. contains (2, 5) and (−2, −1)
31. $m = -\frac{2}{7}$, y-intercept = 4
32. $m = -\frac{3}{2}$, contains (2, −4)
33. $m = 5$, y-intercept = −3
34. contains (3, −1) and (−4, 6)

3-5 Proving Lines Parallel

See pages 151–157.

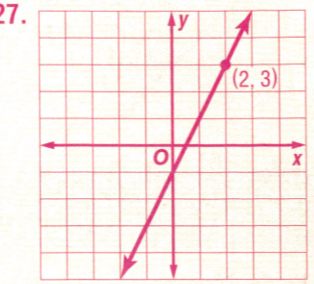
Concept Summary

When lines are cut by a transversal, certain angle relationships produce parallel lines.

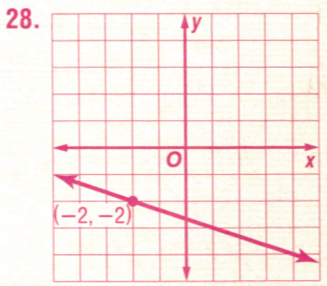
- congruent corresponding angles
- congruent alternate interior angles
- congruent alternate exterior angles
- supplementary consecutive interior angles

Answers

23. neither
24. parallel
25. perpendicular
26. parallel
27.

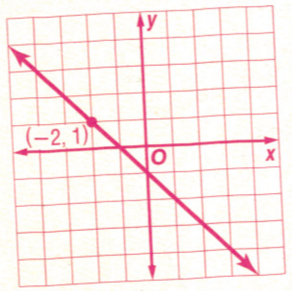


28.

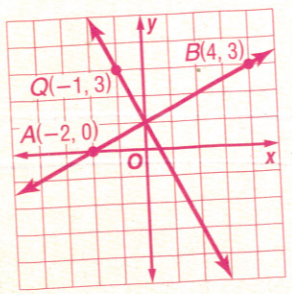


29. $y = 2x - 7$
30. $y = \frac{3}{2}x + 2$
31. $y = -\frac{2}{7}x + 4$
32. $y = -\frac{3}{2}x - 1$
33. $y = 5x - 3$
34. $y = -x + 2$

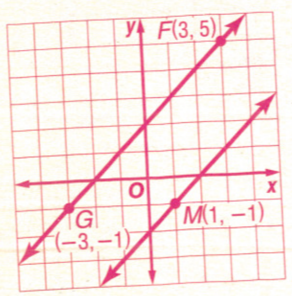
nswers (p. 171)



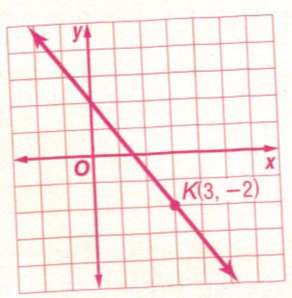
3.



14.



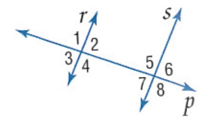
15.



• Extra Practice, see pages 758–760.
• Mixed Problem Solving, see page 784

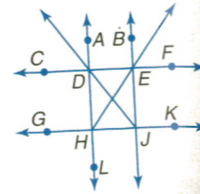
Example If ∠1 ≅ ∠8, which lines if any are parallel?

∠1 and ∠8 are alternate exterior angles for lines *r* and *s*. These lines are cut by the transversal *p*. Since the angles are congruent, lines *r* and *s* are parallel by Theorem 3.5.



Exercises Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.

See Example 1 on page 152.



35. ∠GHL ≅ ∠EJK $\overleftrightarrow{AL} \parallel \overleftrightarrow{BJ}$, alt. ext. ∡ ≅
36. m∠ADJ + m∠DJE = 180 $\overleftrightarrow{AL} \parallel \overleftrightarrow{BJ}$, cons. int. ∡ suppl.
37. $\overline{CF} \perp \overline{AL}$, $\overline{GK} \perp \overline{AL}$ $\overleftrightarrow{CF} \parallel \overleftrightarrow{GK}$, 2 lines ⊥ same line
38. ∠DJE ≅ ∠HDJ $\overleftrightarrow{AL} \parallel \overleftrightarrow{BJ}$, alt. int. ∡ ≅
39. m∠EJK + m∠JEF = 180 $\overleftrightarrow{CF} \parallel \overleftrightarrow{GK}$, cons. int. ∡ suppl.
40. ∠GHL ≅ ∠CDH $\overleftrightarrow{CF} \parallel \overleftrightarrow{GK}$, corr. ∡ ≅

3-6 Perpendiculars and Distance

See pages 159–164.

Concept Summary

- The distance between a point and a line is measured by the perpendicular segment from the point to the line.

Example Find the distance between the parallel lines *q* and *r* whose equations are $y = x - 2$ and $y = x + 2$, respectively.

- The slope of *q* is 1. Choose a point on line *q* such as P(2, 0). Let line *k* be perpendicular to *q* through *P*. The slope of line *k* is −1. Write an equation for line *k*.

$y = mx + b$ — Slope-intercept form
$0 = (-1)(2) + b$ — $y = 0, m = -1, x = 2$
$2 = b$ — Solve for *b*. An equation for *k* is $y = -x + 2$.

- Use a system of equations to determine the point of intersection of *k* and *r*.

$y = x + 2$
$y = -x + 2$
$2y = 4$ — Add the equations.
$y = 2$ — Divide each side by 2.

Substitute 2 for *y* in the original equation.
$2 = -x + 2$
$x = 0$ — Solve for *x*.
The point of intersection is (0, 2).

- Now use the Distance Formula to determine the distance between (2, 0) and (0, 2).

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2-0)^2 + (0-2)^2} = \sqrt{8}$

The distance between the lines is $\sqrt{8}$ or about 2.83 units.

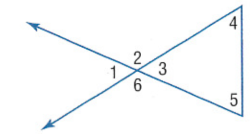
Exercises Find the distance between each pair of parallel lines.

See Example 3 on page 161.

41. $y = 2x - 4$, $y = 2x + 1$ $\sqrt{5}$
42. $y = \frac{1}{2}x$, $y = \frac{1}{2}x + 5$ $\sqrt{20}$

Chapter 3 Practice Test

Vocabulary and Concepts

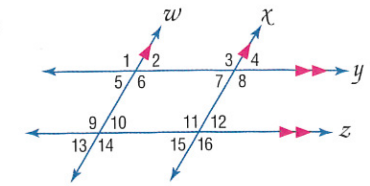


1. Write an equation of a line that is perpendicular to $y = 3x - \frac{2}{7}$. Sample answer: $y = -\frac{1}{3}x + 1$
2. Name a theorem that can be used to prove that two lines are parallel.
3. Find all the angles that are supplementary to ∠1. ∠2, ∠6

2. Sample answer: If alt. int. ∡ are ≅, then lines are ∥.

Skills and Applications

In the figure, m∠12 = 64. Find the measure of each angle.

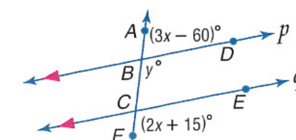


4. ∠8 116
5. ∠13 64
6. ∠7 64
7. ∠11 116
8. ∠3 116
9. ∠4 64
10. ∠9 116
11. ∠5 64

Graph the line that satisfies each condition. 12–15. See margin.

12. slope = −1, contains P(−2, 1)
13. contains Q(−1, 3) and is perpendicular to $\overline{AB}$ with A(−2, 0) and B(4, 3)
14. contains M(1, −1) and is parallel to $\overline{FG}$ with F(3, 5) and G(−3, −1)
15. slope = $-\frac{4}{3}$, contains K(3, −2)

For Exercises 16–21, refer to the figure at the right. Find each value if *p* ∥ *q*.



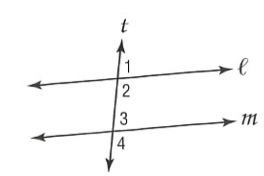
16. *x* 45
17. *y* 105
18. m∠FCE 105
19. m∠ABD 75
20. m∠BCE 75
21. m∠CBD 105

Find the distance between each pair of parallel lines.

22. $y = 2x - 1$, $y = 2x + 9$ $\sqrt{20} \approx 4.47$
23. $y = -x + 4$, $y = -x - 2$ $\sqrt{18} \approx 4.24$

24. **COORDINATE GEOMETRY** Detroit Road starts in the center of the city, and Lorain Road starts 4 miles west of the center of the city. Both roads run southeast. If these roads are put on a coordinate plane with the center of the city at (0, 0), Lorain Road is represented by the equation $y = -x - 4$ and Detroit Road is represented by the equation $y = -x$. How far away is Lorain Road from Detroit Road? about 2.83 mi $2\sqrt{2}$

25. **STANDARDIZED TEST PRACTICE** In the figure at the right, which cannot be true if *m* ∥ ℓ and m∠1 = 73? B



Ⓐ m∠4 > 73
Ⓑ ∠1 ≅ ∠4
Ⓒ m∠2 + m∠3 = 180
Ⓓ ∠3 ≅ ∠1

www.geometryonline.com/chapter_test

Portfolio Suggestion

Introduction Two important terms in this chapter are *parallel* and *perpendicular*. Students used those terms when they explored angles formed by two parallel lines and a transversal.

Ask Students to make an art design that includes parallel lines and a transversal. Have students label angles in their design with letters or color codes and write a key describing the kinds of angle relationships shown. Have students add their art designs to their portfolios.

Chapter 3 Practice Test

Assessment Options

Vocabulary Test A vocabulary test/review for Chapter 3 can be found on p. 174 of the *Chapter 3 Resource Masters*.

Chapter Tests There are six Chapter 3 Tests and an Open-Ended Assessment task available in the *Chapter 3 Resource Masters*.

Chapter 3 Tests

Form	Type	Level	Pages
1	MC	basic	161–162
2A	MC	average	163–164
2B	MC	average	165–166
2C	FR	average	167–168
2D	FR	average	169–170
3	FR	advanced	171–172

MC = multiple-choice questions
FR = free-response questions

Open-Ended Assessment Performance tasks for Chapter 3 can be found on p. 173 of the *Chapter 3 Resource Masters*. A sample scoring rubric for these tasks appears on p. A25.

Unit 1 Test A unit test/review can be found on pp. 181–182 of the *Chapter 3 Resource Masters*.

ExamView® Pro

Use the networkable **ExamView® Pro** to:

- Create **multiple versions** of tests.
- Create **modified** tests for Inclusion students.
- **Edit** existing questions and **add** your own questions.
- Use built-in **state curriculum correlations** to create tests aligned with state standards.
- **Apply** art to your tests from a program bank of artwork.